

## Ratio Problems, including FDP

### Starter

1. Alfie, Billie and Carmen share some money in the ratio 2 : 3 : 7.
- (a) If they share a total of £192, how much do they each get?
  - (b) Billie receives £207. How much do Alfie and Carmen each get?
  - (c) Alfie receives £96. Calculate how much more Carmen gets than Alfie.
  - (d) If Alfie and Carmen receive £324 in total, how much does Billie get?
  - (e) Carmen receives £135 gets more than Alfie, calculate the difference between Billie's and Carmen's amounts.

### Working:

(a)  $2 + 3 + 7 = 12$  parts

$$1 \text{ part} = \frac{192}{12} = 16$$

Alfie receives  $2 \times 16 = £32$

Billie receives  $3 \times 16 = £48$

Carmen receives  $7 \times 16 = £112$

(b) Billie receives £207  $\Rightarrow 3$  parts  $\equiv 207 \quad \therefore 1 \text{ part} \equiv \frac{207}{3} = 69$

Alfie receives  $2 \times 69 = £138$

Carmen receives  $7 \times 69 = £483$

(c) Alfie receives £96  $\Rightarrow 2$  parts  $\equiv 96 \quad \therefore 1 \text{ part} \equiv \frac{96}{2} = 48$

Alfie, Billie and Carmen share some money in the ratio 2 : 3 : 7

Carmen gets 5 parts more than Alfie.

So Carmen gets  $5 \times 48 = £240$

(d) Total parts for Alfie and Carmen =  $2 + 7 = 9$

$$9 \text{ parts} \equiv 324 \quad \therefore 1 \text{ part} \equiv \frac{324}{9} = 36$$

Billie receives  $3 \times 36 = £108$

(e) Alfie : Billie : Carmen is 2 : 3 : 7

Difference between Carmen and Alfie is 5 parts.

This equals

$$5 \text{ parts} \equiv 135 \quad \therefore 1 \text{ part} \equiv \frac{135}{5} = 27$$

Difference between Carmen and Billie is 4 parts.

Difference between Carmen and Billie =  $4 \times 27 = £108$

**E.g. 1** Let the ratio from  $A : B$  be  $3 : 5$  and the ratio between  $B : C$  be  $4 : 11$ .

Writing out the ratios in three columns we get.

$$\begin{array}{l} A : B : C \\ 3 : 5 \\ \underline{\quad 4 : 11} \end{array}$$

Do not write this line

$$A' : B' : C'$$

- (a) What number could link the 5 and the 4 in the middle column? Write this number underneath the 4, in the place of  $B'$ .
- (b) Calculate the correct numbers for  $A'$  and  $C'$  based on your value for  $B'$ . Simplify your ratio where possible.

**Working:** (a) 20

$$\begin{array}{l} A : B : C \\ 3 : 5 \\ \underline{\quad 4 : 11} \\ : 20 : \end{array}$$

- (b) For  $A'$ : from 5 to 20, multiply by 4 so  $A' = 3 \times 4 = 12$   
 For  $C'$ : from 4 to 20, multiply by 5 so  $C' = 5 \times 11 = 55$   
 $A : B : C \equiv 12 : 20 : 55$

**E.g. 2** (a) The ratio from  $R : S$  is  $7 : 2$  and the ratio between  $S : T$  is  $9 : 13$ . Find the ratio  $R : S : T$ .

(b) The ratio from  $X : Y$  is  $8 : 15$  and the ratio between  $X : Z$  is  $7 : 12$ . Find the ratio  $X : Y : Z$ .

**Working:** (a)

$$\begin{array}{l} R : S : T \\ 7 : 2 \\ \underline{\quad 9 : 13} \\ : 18 : \\ 2 \times 9 = 18 \\ \text{For } R': \text{ from 2 to 18, multiply by 9 so } R' = 7 \times 9 = 63 \\ \text{For } T': \text{ from 9 to 18, multiply by 2 so } T' = 2 \times 13 = 26 \\ R : S : T \equiv 63 : 18 : 26 \end{array}$$

(b)

$$\begin{array}{l} X : Y : Z \\ 8 : 15 \\ \underline{\quad 7 : 12} \\ 56 : \quad : \\ 8 \times 7 = 56 \\ \text{For } Y': \text{ from 8 to 56, multiply by 7 so } Y' = 7 \times 15 = 105 \\ \text{For } Z': \text{ from 7 to 56, multiply by 8 so } Z' = 8 \times 12 = 96 \\ X : Y : Z \equiv 56 : 105 : 96 \end{array}$$

**E.g. 3** There are only blue pens, green pens and red pens in a box. The ratio of blue pens to green pens is 2 : 5. The ratio of green pens to red pens is 4 : 1. There are less than 100 pens in the box. What is the greatest possible number of red pens in the box?

**Working:**

$$\begin{array}{r}
 G : B : R \\
 2 : 5 \\
 \quad \quad 4 : 1 \\
 \hline
 \quad \quad : 20 : \\
 4 \times 5 = 20 \\
 \text{For } G': \quad \text{from 5 to 20, multiply by 4 so } G' = 2 \times 4 = 8 \\
 \text{For } R': \quad \text{from 4 to 20, multiply by 5 so } R' = 1 \times 5 = 5 \\
 B : G : R \equiv 8 : 20 : 5 \\
 8 + 20 + 5 = 33 \text{ parts} \\
 1 \text{ part} = \frac{100}{33} = 3.\dot{0}\dot{3} \\
 \text{Red pens} = 5 \times 3.\dot{0}\dot{3} = 15.\dot{1}\dot{5} \\
 \text{The maximum number of red pens is 15.}
 \end{array}$$

**E.g. 4** A grandmother wins £1200 at bingo. She keeps half and gives the remainder to her two children in the ratio of their ages, 36 and 34. Her eldest child shares all the money they received with their children in the ratio of their heights: 86 cm, 97 cm and 120 cm. How much did the smallest child receive? Give your answer to the nearest penny.

**Working:**

$$\begin{array}{l}
 \text{Money to share} = \frac{1200}{2} = 600 \\
 \text{Number of parts} = 46 + 44 = 90 \\
 1 \text{ part} = \frac{600}{80} = 7.5 \\
 \text{Eldest child receives } 36 \times 7.5 = \text{£}270 \\
 \text{Number of parts} = 86 + 97 + 120 = 303 \\
 1 \text{ part} = \frac{270}{303} = \frac{9}{101} \quad \text{use the cancelled fraction for greater accuracy} \\
 \text{Smallest child receives } 86 \times \frac{9}{101} = \text{£}7.66
 \end{array}$$

**From ratios to quantities**

**E.g. 5** Three people have savings in the ratio 4 : 3 : 9.

- (a) Write down two possibilities for the actual savings that each person has.
- (b) Write down how we get possible quantities from the ratio.
- (c) Write down a general form for the quantities using algebra.

**Working:**

- (a) The three people could have £8, £6 and £18 respectively, *...or...* the three people could have £400, £300 and £900 respectively.
- (b) To find a possible quantity, multiply the ratios by the same number.
- (c)  $4x : 3x : 9x$

**E.g. 6** David, Edith and Frankie share some money in the ratio 4 : 9 : 17.

- (a) If Edith and Frankie's combined amount is £748 more David's, find how much they each received.
- (b) Edith gives Frankie a third of her share and is left with £84. Work out how much Frankie now has.
- (c) Edith gives David £130 and they now have the same amount. How much did Frankie receive?
- (d) Edith and Frankie both give David £90, so that David now has twice as much as Edith. How much does Frankie have left? Give your answer to the nearest penny.
- (e) David gives  $\frac{1}{6}$  of her money to charity, Edith gives  $\frac{3}{8}$  and Frankie gives  $\frac{1}{16}$ . If they give £90 in total, how much were they each given? Give your answers to the nearest penny.

**Working:** (a) **Ratio** 4 : 9 : 17  
**Quantities**  $4x : 9x : 17x$   
 Edith + Frankie =  $9x + 17x = 26x$   
 Edith and Frankie's combined amount is £748 more David's  
 $26x - 4x = 748$   
 $22x = 748$   
 $x = \frac{748}{22} = 34$

David receives  $4x = 4 \times 34 = \text{£}136$   
 Edith receives  $9x = 9 \times 34 = \text{£}306$   
 Frankie receives  $17x = 17 \times 34 = \text{£}578$

...Or...

Edith + Frankie =  $9 + 17 = 26$  parts  
 This is 22 parts more than David so  $22 \text{ parts} \equiv 748$   
 $\therefore 1 \text{ part} \equiv \frac{748}{22} = 34$

David receives  $4 \times 34 = \text{£}136$   
 Edith receives  $9 \times 34 = \text{£}306$   
 Frankie receives  $17 \times 34 = \text{£}578$

(b) **Ratio** 4 : 9 : 17  
**Quantities**  $4x : 9x : 17x$   
 $\frac{1}{3}$  of  $9x = 3x$  so  $9x - 3x = 6x = 84$   $x = \frac{84}{6} = 14$   
 Frankie initially received  $17x = 17 \times 14 = 238$   
 Edith gave Frankie  $3x = 3 \times 14 = 42$   
 Frankie now has  $238 + 42 = \text{£}280$

...Or...

Edith gives Frankie a third of her share and is left with £84. Work out how much Frankie now has.

Edith started with 9 parts  
 $\frac{1}{3}$  of 9 = 3 so 6 parts  $\equiv 84$   $\therefore 1 \text{ part} \equiv \frac{84}{6} = 14$

Frankie initially received  $17 \times 14 = 238$   
 Edith gave Frankie  $3 \times 14 = 42$   
 Frankie now has  $238 + 42 = \text{£}280$

(c) **Ratio** 4 : 9 : 17  
**Quantities**  $4x : 9x : 17x$

David and Edith must both have  $\frac{4x + 9x}{2} = 6.5x$   
Edith must give David  $9x - 6.5x = 2.5x$  parts  
So  $2.5x = 130$   $x = \frac{130}{2.5} = 52$   
Frankie received  $17x = 17 \times 52 = \text{£}884$

**...Or...**  
David started with 4 parts; Edith started with 9 parts.  
Edith must give David  $\frac{9 - 4}{2} = 2.5$  parts  
So 2.5 parts  $\equiv 130$   $\therefore 1 \text{ part} \equiv \frac{130}{2.5} = 52$   
Frankie received  $17 \times 52 = \text{£}884$

(d) **Ratio** 4 : 9 : 17  
**Quantities**  $4x : 9x : 17x$

Edith and Frankie both give David  $\text{£}90$   
 $4x + 180 : 9x - 90 : 17x - 90$   
David now has twice as much as Edith  
 $4x + 180 = 2(9x - 90)$

**Solving**  $4x + 180 = 18x - 180$   
 $360 = 14x$   
 $x = \frac{360}{14} = \frac{180}{7}$

Money Frankie has left  $17x - 90 = 17 \times \frac{180}{7} - 90 = \text{£}347.14$

(e) **Ratio** 4 : 9 : 17  
**Quantities**  $4x : 9x : 17x$

Total given away =  $\frac{1}{6} \times 4x + \frac{3}{8} \times 9x + \frac{1}{16} \times 17x = \frac{245}{48}x$

This equals  $\text{£}90$   $\frac{245}{48}x = 90$   
 $x = \frac{90 \times 48}{245} = \frac{864}{49}$

David received  $4 \times \frac{864}{49} = \text{£}70.53$   
Edith received  $9 \times \frac{864}{49} = \text{£}158.69$   
Frankie received  $17 \times \frac{864}{49} = \text{£}299.76$

**Video:** [Ratio - given one quantity](#)  
**Video:** [Ratio - given two ratios](#)

[Solutions to Starter and E.g.s](#)

### Exercise

Worksheet: Ratio and Proportion Problem Solving

Worksheet: AQA Ratio Exam questions