

## Linear Equations involving Fractions

### Starter

1. (Review of last lesson)

Solve: (a)  $2(3x - 4) = 35$  (b)  $5(p + 3) = 7(p - 4) + 53$

**Working:**

(a)  $2(3x - 4) = 35$   
*Expand the brackets*  
 $6x - 8 = 35$   
*Add 8 to both sides*  
 $6x = 43$   
*Divide both sides by 6*  
 $x = \frac{43}{6}$

(b)  $5(p + 3) = 7(p - 4) + 53$   
*Put all p terms on one side*  
 $5(p + 3) - 7(p - 4) = 53$   
*Expand the brackets*  
 $5p + 15 - 7p + 28 = 53$   
*Collect like terms*  
 $-2p + 43 = 53$   
*Subtract 43 from both sides*  
 $-2p = 10$   
*Divide both sides by -2*  
 $p = -5$

2. (Review of last lesson)

The length of a rectangle is three times its width. Its perimeter is 32 m. Find its area.

**Working:**

Let  $w$  be the width of the rectangle.  
 So the length is  $3w$   
*Perimeter of a rectangle*  
 $2(3w + w) = 32$   
*Collect like terms in the bracket*  
 $2 \times 4w = 32$   
 $8w = 32$   
 $w = 4$

So the length =  $3 \times 4 = 12$   
 $\therefore$  area =  $12 \times 4 = 48 \text{ m}^2$ .

**E.g. 1** Solve: (a)  $\frac{x}{5} + 6 = 17$  (b)  $2 = \frac{x}{5} - 7$  (c)  $13 - \frac{2x}{9} = 21$

**Working:**

(a) *Addition before division*  
 $\frac{x}{5} + 6 = 17$   
*Subtract 6 from both sides*  
 $\frac{x}{5} = 11$   
*Multiply both sides by 5*  
 $x = 55$

(b) *Subtraction before division*  
 $2 = \frac{x}{5} - 7$   
*Add 7 to both sides*  
 $9 = \frac{x}{5}$   
*Multiply both sides by 5*  
 $45 = x$   
*Put the unknown on the LHS*  
 $x = 45$

(c) *Addition before division*  
 $13 - \frac{2x}{9} = 21$   
*Subtract 13 from both sides*  
 $-\frac{2x}{9} = 8$   
*Multiply both sides by 9*  
 $-2x = 72$   
*Divide both sides by -2*  
 $x = -36$

**E.g. 2** Solve: (a)  $\frac{2x + 5}{3} = 15$  (b)  $5 = \frac{3x - 1}{7}$  (c)  $\frac{x - 4}{3} + 6 = 8$

**Working:** (a) *Use brackets*  $\frac{(2x + 5)}{3} = 15$   
*Solving: division before brackets*  
*Multiply both sides by 3*  
*Subtract 5 from both sides*  
*Divide both sides by 2*

$$2x + 5 = 45$$

$$2x = 40$$

$$x = 20$$

(b) *Use brackets*  $5 = \frac{(3x - 1)}{7}$   
*Solving: division before brackets*  
*Multiply both sides by 7*  
*Add 1 from both sides*  
*Divide both sides by 2*  
*Put unknown on the LHS*

$$35 = 3x - 1$$

$$36 = 3x$$

$$12 = x$$

$$x = 12$$

(c) *The  $x - 4$  can be considered to be in brackets*  
*Subtraction before division*  $\frac{(x - 4)}{3} + 6 = 8$   
*Subtract 6 from both sides*  
 $\frac{(x - 4)}{3} = 2$   
*Multiply both sides by 3*  
*Add 4 to both sides*

$$x - 4 = 6$$

$$x = 10$$

**E.g. 3** Solve: (a)  $\frac{y - 2}{3} = \frac{y + 4}{5}$  (b)  $\frac{7 - m}{3} = \frac{m + 2}{2}$  (c)  $\frac{4a + 7}{5} = \frac{1 - 8a}{6}$

**Working:** (a) *Cross-multiply*  
*Expand the brackets*  
*Collect like terms*  
*Divide both sides by 2*

$$\frac{y - 2}{3} = \frac{y + 4}{5}$$

$$5(y - 2) = 3(y + 4)$$

$$5y - 10 = 3y + 12$$

$$2y = 22$$

$$y = 11$$

(b) *Cross-multiply*  
*Expand the brackets*  
*Collect like terms*  
*Divide both sides by 5*  
*Put unknown on the LHS*

$$\frac{7 - m}{3} = \frac{m + 2}{2}$$

$$2(7 - m) = 3(m + 2)$$

$$14 - 2m = 3m + 6$$

$$8 = 5m$$

$$\frac{8}{5} = m$$

$$m = \frac{8}{5} = 1\frac{3}{5} = 1.6$$



(c)

$$\frac{4a + 7}{5} = \frac{1 - 8a}{6}$$

*Cross-multiply*  
*Expand the brackets*  
*Collect like terms*

$$6(4a + 7) = 5(1 - 8a)$$

$$24a + 42 = 5 - 40a$$

$$64a = -37$$

*Divide both sides by 64*

$$a = -\frac{37}{64}$$

**E.g. 4** Solve: (a)  $\frac{5}{8}(2p + 5) = \frac{1}{2}(3p - 4)$  (b)  $\frac{5}{7}(m - 2) = \frac{2}{3}(1 - 4m)$

**Working:** (a)

*Create single fractions*

$$\frac{5}{8}(2p + 5) = \frac{1}{2}(3p - 4)$$

*Expand brackets in numerator*

$$\frac{5(2p + 5)}{8} = \frac{1(3p - 4)}{2}$$

*Cross-multiply*  
*Expand the brackets*  
*Collect like terms*

$$2(10p + 25) = 8(3p - 4)$$

$$20p + 50 = 24p - 32$$

$$82 = 4p$$

*Divide both sides by 4*

$$\frac{82}{4} = p$$

*Put unknown on the LHS*

$$p = \frac{82}{4} = \frac{41}{2} = 20\frac{1}{2}$$

(b)

*Create single fractions*

$$\frac{5}{7}(m - 2) = \frac{2}{3}(1 - 4m)$$

*Expand brackets in numerator*

$$\frac{5(m - 2)}{7} = \frac{2(1 - 4m)}{3}$$

*Cross-multiply*  
*Expand the brackets*  
*Collect like terms*

$$3(5m - 10) = 7(2 - 8m)$$

$$15m - 30 = 14 - 56m$$

$$71m = 44$$

*Divide both sides by 71*

$$m = \frac{44}{71}$$

**E.g. 5** Solve: (a)  $\frac{12}{n+1} = \frac{21}{n+4}$  (b)  $\frac{5}{n+3} = \frac{4}{n+5}$  (c)  $\frac{15}{x+4} = \frac{19}{x+3}$

**Working:**

(a)

*Cross-multiply*  
*Expand the brackets*  
*Subtract 21 from both sides*  
*Subtract 12n from both sides*  
*Divide both sides by 9*  
*Rearrange so that n = ...*

$$\begin{array}{r} \frac{12}{n+1} = \frac{21}{n+4} \\ 12(n+4) = 21(n+1) \\ 12n + 48 = 21n + 21 \\ 12n + 27 = 21n \\ 27 = 9n \\ 3 = n \\ n = 3 \end{array}$$

(b)

*Cross-multiply*  
*Expand the brackets*  
*Subtract 4n from both sides*  
*Subtract 25 from both sides*

$$\begin{array}{r} \frac{5}{n+3} = \frac{4}{n+5} \\ 5(n+5) = 4(n+3) \\ 5n + 25 = 4n + 12 \\ n + 25 = 12 \\ n = -13 \end{array}$$

(c)

*Cross-multiply*  
*Expand the brackets*  
*Subtract 15x from both sides*  
*Subtract 76 from both sides.*  
*Divide both sides by 4*  
*Swap to get x on the LHS*

$$\begin{array}{r} \frac{15}{x+4} = \frac{19}{x+3} \\ 15(x+3) = 19(x+4) \\ 15x + 45 = 19x + 76 \\ 45 = 4x + 76 \\ -31 = 4x \\ \frac{31}{4} = x \\ x = -\frac{31}{4} \end{array}$$

**Video:**  
**Video:**

[Linear equations involving fractions](#)  
[Linear equations -- cross multiplication](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

Corbett: [Linear equations involving fractions](#)