Linear Equations involving Fractions

Starter

1. **(Review of last lesson)**
Solve: (a) $2(3x-4)=35$ (b) $5(p+3)=7(p-4)+53$

2. **(Review of last lesson)**
The length of a rectangle is three times its width. Its perimeter is 32 m. Find its area.

Notes

When solving equations involving fractions, it is important to rearrange using SADMIB:

Subtraction
Addition
Division
Multiplication
Indices (i.e. powers)
Brackets

E.g. 1 Solve: (a) $\frac{x}{5}+6=17$ (b) $2=\frac{x}{5}-7$ (c) $13-\frac{2x}{9}=21$

Working: (a)

Addition before division $\frac{x}{5}+6=17$

Subtract 6 ***from both sides*** $\frac{x}{5}=11$

Multiply both sides by 5 $x=55$

Division line as a bracket

With an equation like $\frac{2x+5}{3}=15$, we can consider the division line similar to a bracket.

i.e. $\frac{(2x+5)}{3}=15$

E.g. 2 Solve: (a) $\frac{2x+5}{3}=15$ (b) $5=\frac{3x-1}{7}$ (c) $\frac{x-4}{3}+6=8$

Working: (a)

Use brackets $\frac{(2x+5)}{3}=15$

Solving: division before brackets

Multiply both sides by 3 $2x+5=45$

Subtract 5 from both sides $2x=40$

Divide both sides by 2 $x=20$

Using cross multiplication

Equations involving two fractions equal to each other, can be solved using ***cross-multiplication***.

This means the denominator on each side multiplies the numerator of the other side.

E.g. $\dfrac{x+7}{2} = \dfrac{x-5}{3} \quad \Rightarrow \quad \dfrac{x+7}{2} \times \dfrac{x-5}{3} \quad \Rightarrow \quad 3(x+7) = 2(x-5)$

We can then expand the brackets, collect like terms and solve.

E.g. 3 Solve: (a) $\dfrac{y-2}{3} = \dfrac{y+4}{5}$ (b) $\dfrac{7-m}{3} = \dfrac{m+2}{2}$ (c) $\dfrac{4a+7}{5} = \dfrac{1-8a}{6}$

Working: (a)

$\dfrac{y-2}{3} \times \dfrac{y+4}{5}$

Cross-multiply $5(y-2) = 3(y+4)$

Expand the brackets $5y - 10 = 3y + 12$

Collect like terms $2y = 22$

Divide both sides by 2 $y = 11$

Fractions in front of brackets

When solving equations involving a number in front of a bracket, it is usual to expand the brackets.

However, when the number in front of the bracket is a fraction, this can complicate matters. In such cases, it is better to create a single fraction of the whole expression.

E.g. $\dfrac{5}{8}(2p+5) = \dfrac{5(2p+5)}{8}$

E.g. 4 Solve: (a) $\dfrac{5}{8}(2p+5) = \dfrac{1}{2}(3p-4)$ (b) $\dfrac{5}{7}(m-2) = \dfrac{2}{3}(1-4m)$

Working: (a)

$\dfrac{5}{8}(2p+5) = \dfrac{1}{2}(3p-4)$

Create single fractions $\dfrac{5(2p+5)}{8} = \dfrac{1(3p-4)}{2}$

Expand brackets in numerator $\dfrac{10p+25}{8} \times \dfrac{3p-4}{2}$

Cross-multiply $2(10p+25) = 8(3p-4)$

Expand the brackets $20p + 50 = 24p - 32$

Collect like terms $82 = 4p$

Divide both sides by 4 $\dfrac{82}{4} = p$

Put unknown on the LHS $p = \dfrac{82}{4} = \dfrac{41}{2} = 20\dfrac{1}{2}$

Unknown in the denominator

Unknowns can also appear in the denominator. Can you use a similar method as before to solve these equations?

E.g. 5 Solve: (a) $\frac{12}{n+1} = \frac{21}{n+4}$ (b) $\frac{5}{n+3} = \frac{4}{n+5}$ (c) $\frac{15}{x+4} = \frac{19}{x+3}$

Working: (a) $\frac{12}{n+1} = \frac{21}{n+4}$

Cross-multiply	$12(n+4) = 21(n+1)$
Expand the brackets	$12n + 48 = 21n + 21$
Subtract 21 ***from both sides***	$12n + 27 = 21n$
Subtract $12n$ ***from both sides***	$27 = 9n$
Divide both sides by 9	$3 = n$
Rearrange so that $n = \ldots$	$n = 3$

Video: Linear equations involving fractions
Video: Linear equations -- cross multiplication

Solutions to Starter and E.g.s

Exercise

Corbett: Linear equations involving fractions

Summary

When solving equations requiring two or more steps, we decide the order using SADMIB.

Subtraction
Addition
Division
Multiplication
Indices (i.e. powers)
Brackets

Key skills: Cross-multiplying
Expanding brackets.
Collecting like terms