

Converting Recurring Decimals To Fractions

Starter

- (Review of last lesson)** (a) An integer is divided by 42. Will the answer be a terminating or a recurring decimal? Give a reason for your answer.
(b) An integer is divided by 50. Will the answer be a terminating or a recurring decimal?
(c) The number 210 is divided by a number, x , and the answer is a terminating decimal between 0 and 1. Give an example of what x could be.
- Write the following recurring decimals in dot notation.
(a) 43.888... (b) 0.676767...
(c) 9.463463463... (d) 0.7382382382...
- Convert these recurring decimals from dot notation to normal decimals.
(a) $0.\dot{2}$ (b) $4.\dot{3}7$ (c) $17.\dot{4}61\dot{8}$ (d) $0.93\dot{8}461\dot{5}$
- Write down the decimal equivalent of: (a) $\frac{1}{3}$ (b) $\frac{2}{3}$

Notes

0.333... and 0.666... are examples of **recurring decimals** — they are when the digits are repeated continuously.

Review of dot notation

Dot notation is used to make the writing of recurring decimals quicker.

E.g. $2.555... = 2.\dot{5}$

Two dots can be used to show that more than one digit is being repeated.

E.g. $0.\dot{7}\dot{2} = 0.727272...$

When the two dots are separated, the digits in between the dots are also repeated.

E.g. $8.\dot{6}1\dot{4} = 8.614614614...$

E.g. $0.52\dot{3}18\dot{9} = 0.52318931893189...$

Converting recurring decimals to fractions

A recurring decimal is truncated when entered into a calculator so if a recurring decimal is used in a calculation it introduces a rounding error. To avoid this we can convert the recurring decimal to a fraction.

To convert a recurring decimal to a fraction, two recurring decimals with the same recurring part must be subtracted from one another.

E.g. 1 Express $0.\dot{4}$ as a fraction.

Working: $0.\dot{4} = 0.444\dots$

Let $x = 0.444\dots$

Multiply by 10: $10x = 4.444\dots$

We now have two decimals with the same recurring part:

$$x = 0.444\dots$$

$$10x = 4.444\dots$$

When the first equation is subtracted from the second, the recurring part will disappear.

Subtracting: $10x - x = 4.444\dots - 0.444\dots$

$$9x = 4$$

$$x = \frac{4}{9}$$

E.g. 2 Express $0.\dot{7}$ as a fraction.

When there are **two recurring digits**, multiplying by 10 won't be enough.

For example, let $x = 0.\dot{9}\dot{3} = 0.939393\dots$

Then $10x = 9.39393\dots$

Comparing the recurring parts we see that they are not the same:

$$x = 0.939393\dots$$

$$10x = 9.39393\dots$$

E.g. 3 Express $0.\dot{9}\dot{3}$ as a fraction in its lowest terms.

Working: $0.\dot{9}\dot{3} = 0.939393\dots$

Let $x = 0.939393\dots$

Multiply by 100: $100x = 93.939393\dots$

Subtracting: $100x - x = 93.939393\dots - 0.939393\dots$

$$99x = 93$$

$$x = \frac{93}{99} = \frac{31}{33}$$

E.g. 4 Express these recurring decimals as fractions in their lowest terms.

(a) $0.\dot{1}\dot{6}$

(b) $0.\dot{7}\dot{2}$

(c) $0.\dot{4}5\dot{8}$

What happens when there is a non-recurring digit after the decimal point?

For example, $0.8\dot{5} = 0.8555\dots$ or $0.94\dot{6} = 0.94666\dots$

In such situation, we need to multiply by 10, 100 so that the **recurring part is next to the decimal point**.

E.g. 5 Express $0.8\dot{5}$ as a fraction in its lowest terms.

Working: $0.8\dot{5} = 0.8555\dots$
Let $x = 0.8555\dots$

Multiply by 10 to get recurring part next to decimal point:

$$10x = 8.555\dots$$

Multiply $x = 0.8555\dots$ by 100: $100x = 85.555\dots$

Subtracting: $100x - 10x = 85.555\dots - 8.555\dots$

$$90x = 77$$

$$x = \frac{77}{90}$$

E.g. 6 Express these recurring decimals as fractions in their lowest terms.

(a) $0.3\dot{2}$

(b) $0.94\dot{6} = 0.94666\dots$

(c) $0.49\dot{5}$

Video: [Recurring decimals to fractions](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p10 E1.1 Qu 1-2

A*-G class textbook: p10 E1.1 Qu 1-2

9-1 homework book: p4 E1.1 Qu 1-3

A*-G homework book: p3 E1.1 Qu 1-3

Summary

Converting recurring decimals to fractions:

To convert a recurring decimal to a fraction, two recurring decimals with the same recurring part must be subtracted from one another.

When there is a non-recurring digit after the decimal point, multiply by 10, 100 so that the **recurring part is next to the decimal point**.