Standard deviation from frequency tables

Starter

1. ***(Review of last lesson)*** Find the interquartile range for these data values:

Weights of students (kg)

4	3 6
5	1 2 5 7 7
6	3 4 7 8
7	4 7
8	1

Key: 4 | 3 means 43

2. Find the mean for the data in this frequency table (you may wish to draw another column and another row to aid the calculation).

x	**Frequency,** f
2	8
3	11
4	13
5	9

3. Using your answer to question 2, write down a formula involving Σ for the mean in terms of f and x.

Notes

Formula for standard deviation from a frequency table

Remember: $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

	Individual data values	**Frequency Table**
Original formula	$\sigma = \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}}$	$\sigma = \sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$
Formula to use	$\sigma = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$	$\sigma = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

E.g. 1 Using the table below, find the standard deviation for the data from question 2 of the starter.

x	**Frequency,** f	$f \times x$	x^2	$f \times x^2$
2	8	$8 \times 2 = 16$		
3	11	$11 \times 3 = 33$		
4	13	$13 \times 4 = 52$		
5	9	$9 \times 5 = 45$		
	$\Sigma f = 41$	$\Sigma fx = 146$		

Working:

x	**Frequency,** f	$f \times x$	x^2	$f \times x^2$
2	8	$8 \times 2 = 16$	4	$8 \times 4 = 32$
3	11	$11 \times 3 = 33$	9	$11 \times 9 = 99$
4	13	$13 \times 4 = 52$	16	$13 \times 16 = 208$
5	9	$9 \times 5 = 45$	25	$9 \times 25 = 225$
	$\Sigma f = 41$	$\Sigma fx = 146$		$\Sigma fx^2 = 564$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} = \sqrt{\frac{564}{41} - \left(\frac{146}{41}\right)^2} = 1.04 \text{ (3 s.f.)}$$

E.g. 2 The number of mobile phones for a group of families is given below:

Number of mobile phones	0	1	2	3	4	5
Number of families	6	9	17	16	12	8

Calculate the mean and standard deviation for this data.

Grouped data

E.g. 3 The waiting times for patients at a doctor's surgery are given below. Calculate estimates for the mean and standard deviation.

Time (minutes)	**Number of patients**
$0 \leq t < 2$	3
$2 \leq t < 4$	8
$4 \leq t < 8$	15
$8 \leq t < 20$	9

Using your calculator

Menu >> 6:Statistics >> 1:1—Variable >> [Enter the data values in the left column and the frequency of each data value in the right column] >> AC >> OPTN >> 2:1—Variable Calc

$\bar{x}$ mean

σx standard deviation

N.B. To get a frequency column to appear, press:
Shift >> Menu >> Down arrow >> 3: Statistics >> Press 1 to turn Frequency On

E.g. 4 The heights of students in a school year are given below. Use the statistical functions of your calculator to find the mean and standard deviation.

Height (m)	Number of students
140–	3
146–	8
152–	15
161 – 170	9

Finding the standard deviation of big numbers with only a basic calculator

When data involves large x—values, the calculation with only a basic calculator can become laborious. However, subtracting a number from each value the calculation can be made much easier — this is a basic ***method of coding***.

Change	Effect on mean	Effect on standard deviation
Add k to each data value	Add k	No effect
Multiply each data value by k	Multiply by k	Multiply by k

E.g. 5 In a golf competition, the number of shots taken for the round are given below:

Number of shots	75	76	77	78	79
Number of players	4	9	5	11	7

By using only the basic functions of a calculator, calculate the mean and standard deviation for this data.

Hint: use the method of coding by subtracting 77 from the player's scores.

Working: Subtract 77 from each value.

Number of shots, x	75	76	77	78	79	
Number of players, f	4	9	5	11	7	$\Sigma f = 36$
$x - 77$	-2	-1	0	1	2	
$f(x-77)$	-8	-9	0	11	14	$\Sigma f(x-77) = 8$
$(x-77)^2$	4	1	0	1	4	
$f(x-77)^2$	16	9	0	11	28	$\Sigma f(x-77)^2 = 64$

Remember to add 77 to get the value for the mean.

$$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{8}{36} + 77 = 77.\dot{2}$$

The standard deviation is unchanged by subtracting a number.

$$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} = \sqrt{\frac{64}{36} - \left(\frac{8}{36}\right)^2} = 1.31 \text{ (3 s.f.)}$$

Video: Estimating the median, quartiles, percentiles from a grouped frequency table

Video: Mean and standard deviation from grouped frequency table

Exam questions: Estimating median, quartiles, percentiles from grouped data

Solutions to Starter and E.g.s

Exercise

p340 16C Qu 1i, 2i, 3-7, (8-9 red), 10

Summary

	Individual data values	**Frequency Table**
Original formula	$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$	$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$
Formula to use	$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$	$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

Using your calculator:

Menu >> 6:Statistics >> 1:1—Variable >> [Enter the data values in the left column and the frequency of each data value in the right column] >> AC >> OPTN >> 2:1—Variable Calc

$\bar{x}$ mean

σx standard deviation