L6 Maths January Exam Teacher X 25-26 SOLUTIONS [55]

1

(a)		$2x^{-\frac{1}{4}}$	**B1** **[1]**	**1.1**	Correct expression	Or $2x^{-0.25}$ Could be implied by $p = 2$, $q = -0.25$
(b)		$125\ldots$ $\ldots x^{\frac{9}{2}}$	**B1** **B1** **[2]**	**1.1** **1.1**	Obtain 125 Obtain $x^{\frac{9}{2}}$	Could be implied by $p = 125$ Allow $4\frac{1}{2}$ or 4.5 Could be implied by $q = 4.5$ Allow **SC B1** if 0 scored but $x\sqrt{x}$ is seen as $x^{\frac{3}{2}}$ oe
(c)		$4\ldots$ $\ldots x^4$	**B1** **B1** **[2]**	**1.1** **1.1**	Obtain 4 Obtain x^4	Could be implied by $p = 4$ Could be implied by $q = 4$ Allow **SC B1** if 0 scored but $\sqrt{16x^8}$ seen
(d)		$\left(27x^6\right)^{\frac{1}{3}} = 3x^2$ $x^5 \times 3x^2 = 3x^7$	**M1*** **M1dep*** **A1** **[3]**	**1.1** **1.1** **1.1**	Obtain either ax^2 or $3x^b$ Correctly combine x^5 and their $3x^2$ Obtain correct answer	 Obtain either ax^7 following their a, or $3x^{(5+b)}$ following their b Could be implied by $p = 3$, $q = 7$

2.

(a)		**DR** $x^2 - 11 = 25$ $[x^2 = 36] \Rightarrow x = \pm 6$	**M1** **A1** **[2]**	**1.1** **1.1**	Square both sides
(b)		**DR** $(y^3)^2 + 7y^3 - 8 = 0$ or $u = y^3$ $(y^3 + 8)(y^3 - 1) = 0$ or $(u + 8)(u - 1) = 0$ $y^3 = -8$ or 1 or $u = -8$ or 1 $y = -2$ or 1	**M1** **B1** **A1** **A1** **[4]**	**1.1** **1.1** **1.1** **1.1**	soi (implied by B1) Allow other letters but not $y = y^3$ Correct working or use of formula to solve a quadratic in u, y^3 oe. Condone omission of $= 0$ and sign errors for this mark (NB answers only without working scores 0/4)

3.

11 (a)	Uses $t = 0, V = 20\,000 \Rightarrow 20\,000 = 1500 + A\mathrm{e}^{0}$	M1
	$A = 18\,500$	A1
	$t = 2.5, V = 12\,000 \Rightarrow 12\,000 = 1500 + 18\,500\mathrm{e}^{-k\times 2.5}$ $\Rightarrow 10500 = 18500\mathrm{e}^{-k\times 2.5} \Rightarrow k = \ldots \left(= -\frac{2}{5}\ln\frac{21}{37} = \text{awrt } 0.227 \right)$	dM1
	$V = 1500 + 18500\,\mathrm{e}^{-0.227t}$	A1
		(4)
(b)	Achieves $\left(\frac{\mathrm{d}V}{\mathrm{d}t} =\right) -kA\mathrm{e}^{-kt}$ or $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	B1
	Substitutes $A\mathrm{e}^{-kt} = V - 1500$ into $\left(\frac{\mathrm{d}V}{\mathrm{d}t} =\right) -kA\mathrm{e}^{-kt}$ or $"18500"\mathrm{e}^{"-0.227"t} = V - 1500$ into $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	M1
	Rate of change in value of car is $\left(\frac{\mathrm{d}V}{\mathrm{d}t} =\right) -k(V - 1500)$ or $"-0.227"(V - 1500)$ *	A1*
		(3)

4.

DR $3(x-2)(2x+1) \leqslant 9$	**M1***	**3.1a**	Setting up an inequality or equation with the correct three lengths and 9	Allow any inequality sign or equals
$2x^2 - 3x - 5 \ (\leqslant 0)$	**M1dep***	**1.1**	Forming a three-term quadratic expression with all terms on the same side	Allow a single sign slip when simplifying
$(2x-5)(x+1) \ (\leqslant 0)$ $x = \frac{5}{2}, \quad x = -1$	**B1**	**1.1**	Solving their three-term quadratic correctly by factorisation (oe)	If using factorisation then when expanded x^2 term and at least one other term correct for this mark Dependent on both **M** marks
Either $\lambda \leqslant x \leqslant \frac{5}{2}$ **or** $\lambda \leqslant x$, $x \leqslant \frac{5}{2}$	**A1**	**2.2a**	Dependent on the first two **M** marks – allow strict inequalities for this mark or a mixture of strict and non-strict – can be implied by correct answer	Where $\lambda = -1, 0$ or 2
$\left(2, \frac{5}{2}\right]$	**A1** **[5]**	**2.5**	cao	Dependent on both **M** marks

5.

Obtains gradient of tangent $= \frac{1}{9}$	1.1b	B1	Gradient of tangent $= \frac{1}{9}$ Gradient of curve $= 3e^{3x}$ $3e^{3x} = \frac{1}{9}$ $e^{3x} = \frac{1}{27}$ $3x = \ln\frac{1}{27}$ $x = \ln\frac{1}{3}$
Recalls gradient function $= 3e^{3x}$	1.2	B1	
Equates their gradient function to their numerical tangent gradient	3.1a	M1	
Uses natural log correctly with their equation of the form $ke^{3x} = m$ to obtain an equation without e	1.1a	M1	
Obtains $x = \ln\frac{1}{3}$ ACF ISW Accept AWRT –1.1	1.1b	A1	

6.

(a)		**DR** $\overrightarrow{OP} = \mathbf{a} + \overrightarrow{AP}$ or $\mathbf{c} + \mathbf{a} + \overrightarrow{BP}$ $\overrightarrow{OP} = \mathbf{a} + \frac{1}{3}\mathbf{c}$ $\overrightarrow{OQ} = \frac{3}{4}\overrightarrow{OP} = \frac{3}{4}\left(\mathbf{a} + \frac{1}{3}\mathbf{c}\right)$ $= \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{c}$ **AG**	**M1** **A1** **M1** **A1** **[4]**	**3.1a** **1.1** **1.1** **1.1**	soi If labelled then must be correctly identified as $\overrightarrow{OP}$ Use of $\overrightarrow{OQ} = \frac{3}{4}\overrightarrow{OP}$ with their $\overrightarrow{OP}$ soi, may use $\overrightarrow{OQ} = 3\overrightarrow{QP}$ Must see an intermediate step, as a minimum $\frac{3}{4}\left(\mathbf{a} + \frac{1}{3}\mathbf{c}\right)$oe
(b)		**DR** $\overrightarrow{AQ} = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{c} - \mathbf{a} \left[= \frac{1}{4}\mathbf{c} - \frac{1}{4}\mathbf{a}\right]$ $\left[\overrightarrow{AC} = \mathbf{c} - \mathbf{a}\right]$ $\Rightarrow \overrightarrow{AQ} = \frac{1}{4}\overrightarrow{AC}$ **or** $\overrightarrow{AC}$ is a [scalar] multiple of $\overrightarrow{AQ}$ **or** $\overrightarrow{AQ}$ is parallel to $\overrightarrow{AC}$ **or** $\overrightarrow{AC} = 4\overrightarrow{AQ}$ Hence Q lies on AC **AG**	**M1** **A1** **A1**	**1.1** **2.1** **2.2a**	Or $\overrightarrow{QC}\left[= -\overrightarrow{OQ} + \overrightarrow{OC}\right] = -\frac{3}{4}\mathbf{a} - \frac{1}{4}\mathbf{c} + \mathbf{c}\left[= \frac{3}{4}\mathbf{c} - \frac{3}{4}\mathbf{a}\right]$ oe, e.g. $\overrightarrow{QC} = \frac{3}{4}\overrightarrow{AC}$ may see 'Q,A,C colinear' but must be justified. Correct conclusion, dep $\overrightarrow{AQ} = \frac{1}{4}\overrightarrow{AC}$ oe seen www Other methods may be seen, e.g. using $\overrightarrow{CQ}$ or $\overrightarrow{QA}$
		Alternative method $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{c}$ is of the form $\alpha\mathbf{a} + (1-\alpha)\mathbf{c}$ Hence Q lies on AC	**M2** **A1**		
			[3]		

7.

$x^2-3px+5q+4=0$	
$(3p)^2-4(5q+4)$	B1
$9p^2-20q-16=0$ $(3p+4)(3p-4)=20q$	M1
$q=\frac{1}{20}(3p+4)(3p-4)$ *	A1*
	(3)

8.

$2\log_3(x+1)=1+\log_3(x+7)$	
$2\log_3(x+1)=\log_3(x+1)^2$	B1
$\log_3\frac{(x+1)^2}{x+7}=1$	M1
$\frac{(x+1)^2}{x+7}=3$	A1
$(x+1)^2=3(x+7)\Rightarrow x^2-x-20=0\Rightarrow x=...$	M1
$x=5$ only	A1
	(5)

9.

10(a)	Obtains $\ln Ae^{-kt} = \ln(21 \pm \theta)$ or $\ln e^{-kt} = \ln\frac{21 \pm \theta}{A}$ or $-kt = \ln\frac{21 \pm \theta}{A}$	3.1a	M1	$\theta = 21 - Ae^{-kt}$ $Ae^{-kt} = 21 - \theta$ $\ln Ae^{-kt} = \ln(21 - \theta)$ $\ln A + \ln e^{-kt} = \ln(21 - \theta)$ $\ln A - kt = \ln(21 - \theta)$
	Obtains $\ln Ae^{-kt} = \ln A + \ln e^{-kt}$ or $\ln\frac{21 \pm \theta}{A} = \ln(21 \pm \theta) - \ln A$	1.1a	M1	
	Completes reasoned argument to obtain $\ln(21 - \theta) = -kt + \ln A$ AG	2.1	R1	
	Subtotal		**3**	
10(b)(i)	States $\ln A = 3.676$	3.4	M1	$\ln A = 3.676$ $A = e^{3.676} = 39.5$
	Obtains AWRT 39.5	1.1b	A1	
	Subtotal		**2**	
10(b)(ii)	Obtains $\pm\frac{3.676}{19.98}$ **Or** Substitutes their value of A or $\ln A = 3.676$, $t = 19.98$ and $\ln(21 - \theta) = 0$ to form an equation for k	3.4	M1	$-k = -\frac{3.676}{19.98}$ $k = 0.1839...$ $= 0.184$
	Obtains AWRT $k = 0.184$	1.1b	A1	
	Subtotal		**2**	
10(b)(iii)	Uses the model $\theta = 21 - Ae^{-kt}$ with their A or $\ln A = 3.676$ and $t = 0$ **Or** Equates $\ln(21 - \theta)$ to 3.676	3.4	M1	$\theta = 21 - 39.5e^{0}$ $= -18.5\,^{\circ}C$
	Obtains AWRT $-18.5\,^{\circ}C$ Condone missing units CAO	1.1b	A1	
	Subtotal		**2**	