L6 Further Mathematics End of Year

Paper 1 (Teacher X)

May 2025

2024-2025

Duration: **1 hour 30 minutes**

Total number of marks: **72**

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Relevant information from the formula booklet is included prior to each section of questions.

The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

Pure

1.

A reflection is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

State the equation of the line of invariant points.

$x = 0$ $\quad$ $y = 0$ $\quad$ $y = x$ $\quad$ $y = -x$

[1 mark]

2.

Use a matrix method to determine the solution of the following simultaneous equations. **[4]**

$$\begin{aligned} 2x - 3y + z &= 1 \\ x - 2y - 4z &= 40 \\ 5x + 6y - z &= 61 \end{aligned}$$

3.

The vectors **a**, **b**, and **c** are such that $\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{a} \times \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

Work out $(\mathbf{a} - 4\mathbf{b} + 3\mathbf{c}) \times (2\mathbf{a})$

[4 marks]

4.

A transformation T is represented by the matrix $\mathbf{N} = \begin{pmatrix} a & 4 & 2 \\ 5 & 1 & 0 \\ 3 & 6 & 3 \end{pmatrix}$, where a is a constant.

(a) Find $\mathbf{N}^2$ in terms of a. **[3]**

(b) Find det **N** in terms of a. **[2]**

The value of a is 13 to the **nearest integer**.

A shape S_1 has volume 11.6 to 1 decimal place. Shape S_1 is mapped to shape S_2 by the transformation T.

A student claims that the volume of S_2 is less than 400.

(c) Comment on the student's claim. **[3]**

5.

The drainage system for a sports field consists of underground pipes.

This situation is modelled with respect to a fixed origin O.

According to the model,

- the surface of the sports field is a plane with equation $z = 0$
- the pipes are straight lines
- one of the pipes, P_1, passes through the points $A(3, 4, -2)$ and $B(-2, -8, -3)$
- a different pipe, P_2, has equation $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+1}{-2}$
- the units are metres

(a) Determine a vector equation of the line representing the pipe P_1

(2)

(b) Determine the coordinates of the point at which the pipe P_1 meets the surface of the playing field, according to the model.

(2)

Determine, according to the model,

(c) the acute angle between pipes P_1 and P_2, giving your answer in degrees to 3 significant figures,

(3)

6.

The matrices **A** and **B** are defined as follows.

$$\mathbf{A} = \begin{bmatrix} p-2 & p-1 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 2p-1 \\ 0 & 4-p \end{bmatrix}$$

Find the values of p such that **A** and **B** are commutative under matrix multiplication.

Fully justify your answer.

[4 marks]

7.

You are given that $\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ where a is a constant.

Prove by induction that $\mathbf{A}^n = \begin{pmatrix} 1 & an \\ 0 & 1 \end{pmatrix}$ for all integers $n \geqslant 1$. **[5]**

8.

The matrix **C** is defined by

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}$$

Prove that the transformation represented by **C** has no invariant lines of the form $y = kx$

[4 marks]

Mechanics

9.

A motorbike and its rider, together denoted by M, have a combined mass of 360 kg. The resistive force experienced by M when it is in motion is modelled as being proportional to the speed it is moving at. All motion of M is on a straight horizontal road.

It is found that with the engine of the motorbike working at a rate of 12 kW, the maximum constant speed that M can move at is $10\,\text{m s}^{-1}$.

Determine the speed of M such that with the engine working at a rate of 12 kW the acceleration of M is $1.5\,\text{m s}^{-2}$. **[7]**

10.

In this question you may assume that if x and y are any physical quantities then $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] = \left[\frac{y}{x}\right]$.

A machine drives a piston of mass m into a vertical cylinder. The equation below is suggested to model the power developed by the machine, P, while it is not doing any other external work.

$$P = k_1 mv\frac{\mathrm{d}v}{\mathrm{d}t} + k_2 mgv + k_3 E$$

in which

- v is the velocity of the piston at a given time,
- g is the acceleration due to gravity,
- E is the **rate** at which heat energy is lost to the surroundings,
- k_1, k_2 and k_3 are dimensionless constants.

Determine whether the equation is dimensionally consistent. Show **all** the steps in your argument.

[6]

11.

A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 1.8 m. The other end of the string is attached to a fixed point, O, on a smooth horizontal plane. Initially, P is moving with a constant speed of $12\,\text{m s}^{-1}$ in a horizontal circle with O as its centre.

(a) (i) Find the magnitude of the acceleration of P. **[1]**

(ii) State the direction of the acceleration of P. **[1]**

A force is now applied to P in such a way that its angular velocity increases. At the instant that the angular velocity reaches $8\,\text{rad s}^{-1}$, the string breaks.

(b) (i) Find the speed with which P is moving at the instant that the string breaks. **[1]**

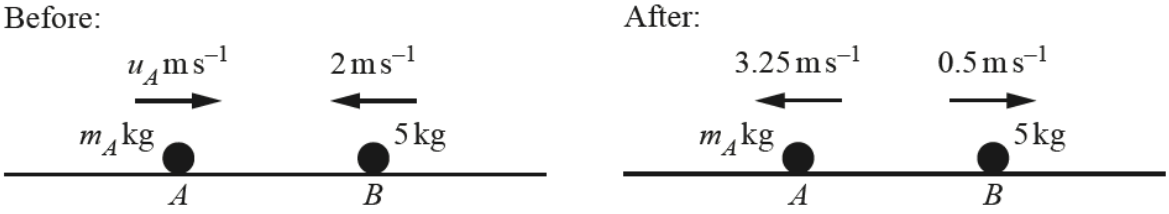
(ii) Find the tension in the string at the instant that the string breaks. **[2]**

After the string has broken P starts to move directly up a smooth slope which is fixed to the plane and inclined at an angle $\theta°$ above the horizontal. Particle P moves a distance of 20 m up the slope before coming to instantaneous rest.

(c) Use an energy method to determine the value of θ. **[3]**

12.

Two particles, A of mass m_A kg and B of mass 5 kg, are moving directly towards each other on a smooth horizontal floor. Before they collide they have speeds $u_A\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively. Immediately after they collide the direction of motion of each particle has been reversed and A and B have speeds $3.25\,\text{m s}^{-1}$ and $0.5\,\text{m s}^{-1}$ respectively (see diagram). The coefficient of restitution between A and B is 0.75.



(a) Determine the value of m_A and the value of u_A. **[5]**

(b) Show that approximately 41% of the kinetic energy of the system is lost in this collision. **[3]**

After the collision between A and B, B goes on to collide directly with a third particle C of mass 3 kg which is travelling towards B with a speed of $5.5\,\text{m s}^{-1}$. The coefficient of restitution between B and C is denoted by e.

(c) Given that, after B and C collide, there are no further collisions between A, B and C determine the range of possible values of e. **[6]**